Imperial College London

E 2.5
Signals & Linear Systems

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Aims and Objectives

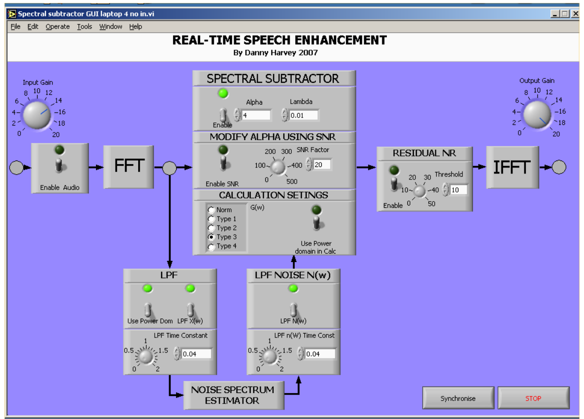
- By the end of the course, you would have understood:
  - Basic signal analysis (mostly continuous-time)
  - Basic system analysis (also mostly continuous systems)
  - Time-domain analysis (including convolution)
  - Laplace Transform and transfer functions
  - Fourier Series (revision) and Fourier Transform
  - Sampling Theorem and signal reconstructions
  - Basic z-transform

About the course

- Lectures - around 9 weeks (15-17 hours)
- Problem Classes – 1 hr per week
- Official Hours – 2 hrs per week (taken by Dr Naylor)
- Assessment – 100% examination in June
- Handouts in the form of PowerPoint slides
- Text Book
  - ***B.P. Lathi, "Linear Systems and Signals", 2nd Ed., Oxford University Press (~£36)***

A demonstration …..

- This is what you will be able to do in your 3rd year (helped by this course)
- You will be able to design and implement a NOISE CANCELLING system



Lecture 1

Basics about Signals

(**Lathi 1.1-1.5**)

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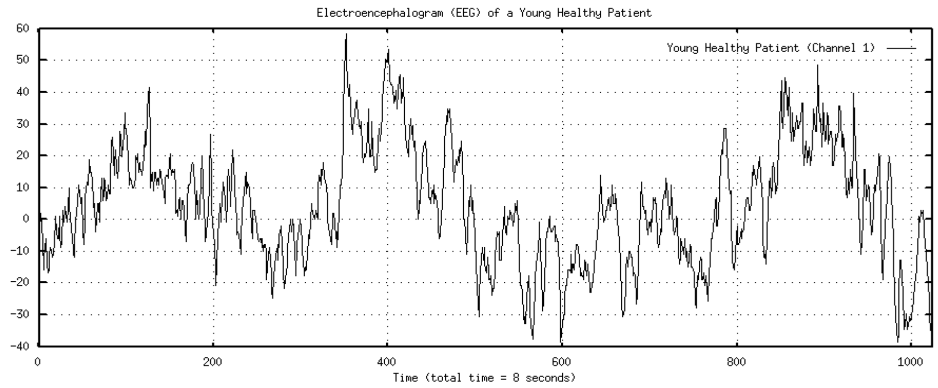
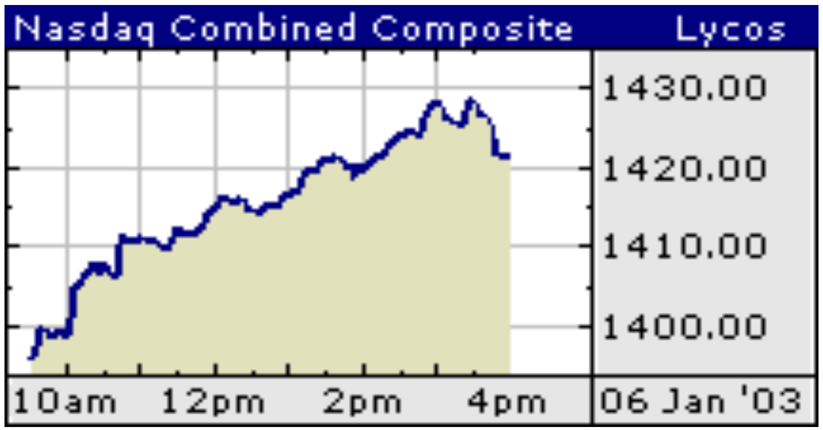
Examples of signals (1)

- Electroencephalogram (EEG) signal (or brainwave)



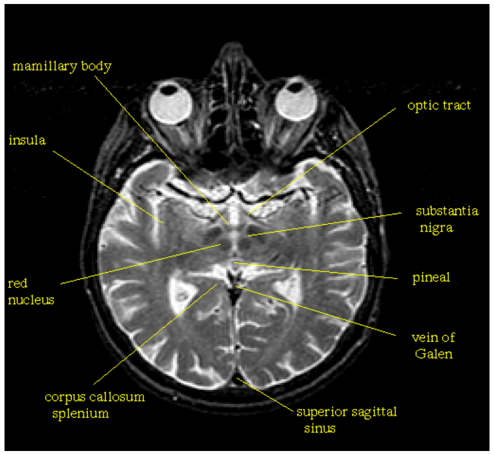
Examples of signals (2)

- Stock Market data as signal (time series)



Examples of signals (3)

- Magnetic Resonance Image (MRI) data as 2-dimensional signal



Size of a Signal *x(t)* (1)

- Measured by signal energy E_x:

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt$$

- Generalize for a complex valued signal to:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

- Energy must be finite, which means

$$\text{signal amplitude} \to 0 \text{ as } |t| \to \infty$$

Lathi Section 1.1

L1.1

Size of a Signal *x(t)* (2)

- If amplitude of *x(t)* does not → 0 when t → ∞, need to measure power *Px* instead:

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

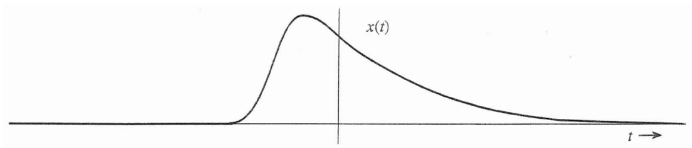
- Again, generalize for a complex valued signal to:

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\,dt$$

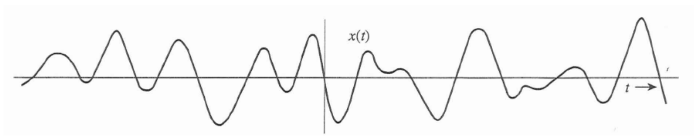
L1.1

Size of a Signal *x(t)* (3)

- Signal with finite energy (zero power)



- Signal with finite power (infinite energy)



L1.1

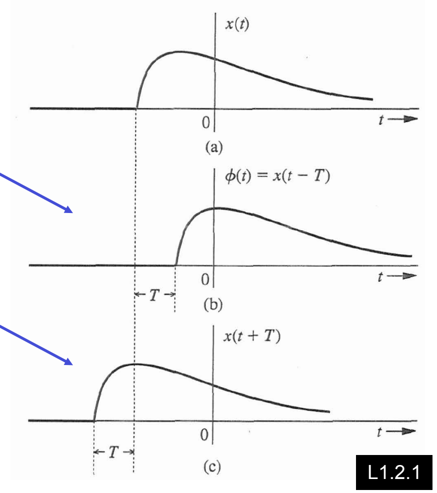
Useful Signal Operations –Time Shifting (1)

- Signal may be delayed by time T:

$$\phi(t+T) = x(t)$$

- or advanced by time T:

$$\phi(t-T) = x(t)$$



L1.2.1

Useful Signal Operations –Time Scaling (2)

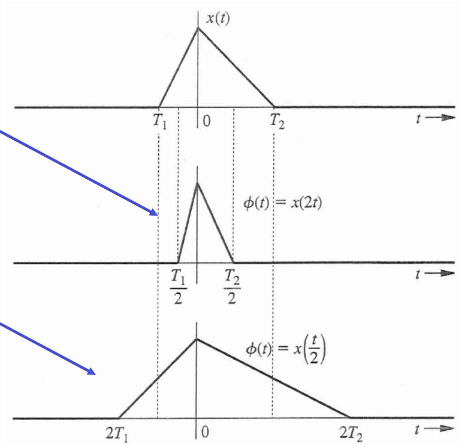
- Signal may be compressed in time (by a factor of 2):

$$\phi\left(\frac{t}{2}\right) = x(t)$$

- or expanded in time (by a factor of 2):

$$\phi(2t) = x(t)$$

- Same as recording played back at twice and half the speed respectively



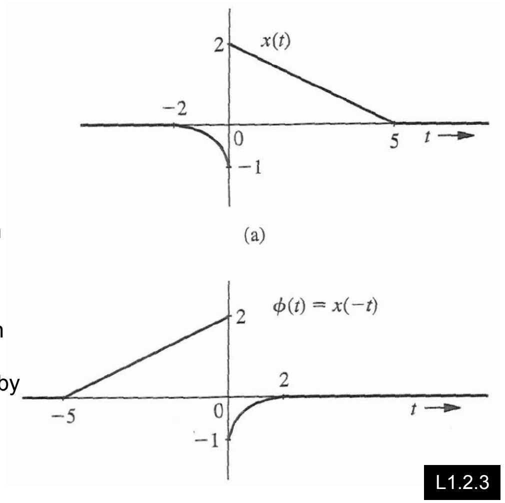
L1.2.2

Useful Signal Operations –Time Reversal (3)

- Signal may be reflected about the vertical axis (i.e. time reversed):

$$\phi(t) = x(-t)$$

- We can combine these three operations.
- For example, the signal x(2t - 6) can be obtained in two ways;
  - Delay x(t) by 6 to obtain x(t - 6), and then time-compress this signal by factor 2 (replace t with 2t) to obtain x (2t - 6).
  - Alternately, time-compress x (t) by factor 2 to obtain x (2t), then delay this signal by 3 (replace t with t - 3) to obtain x (2t - 6).



L1.2.3

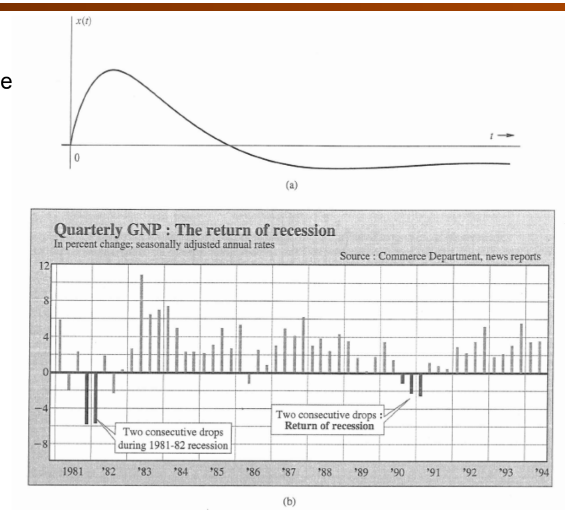
Signals Classification (1)

- Signals may be classified into:
  1. Continuous-time and discrete-time signals
  2. Analogue and digital signals
  3. Periodic and aperiodic signals
  4. Energy and power signals
  5. Deterministic and probabilistic signals
  6. Causal and non-causal
  7. Even and Odd signals

L1.3

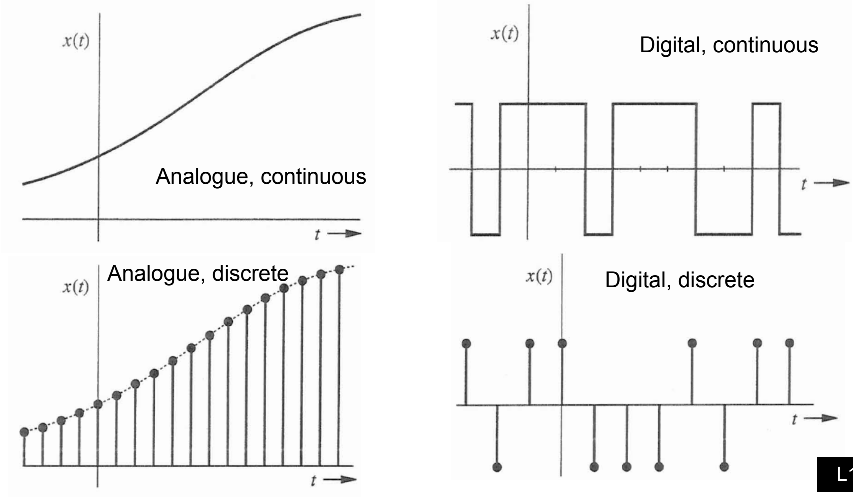
Signal Classification (2) – Continuous vs Discrete

- Continuous-time
- Discrete-time



L1.3

Signal Classification (3) – Analogue vs Digital



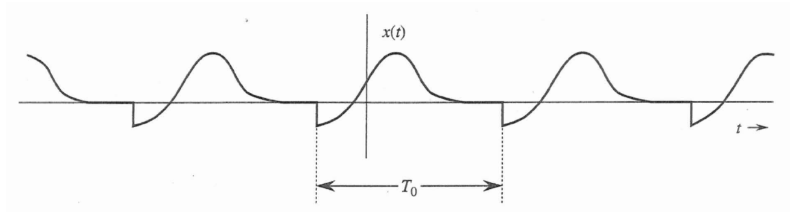
L1.3

Signal Classification (4) – Periodic vs Aperiodic

- A signal *x(t)* is said to be periodic if for some positive constant T_o

$$x(t) = x(t + T_0) \qquad \text{for all } t$$

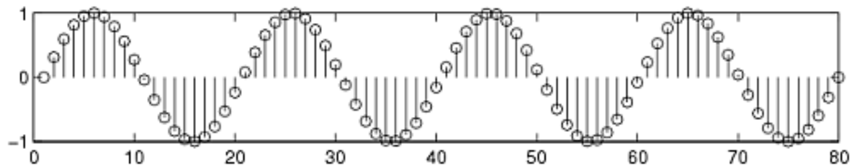
- The smallest value of T_o that satisfies the periodicity condition of this equation is the *fundamental period* of x(t).



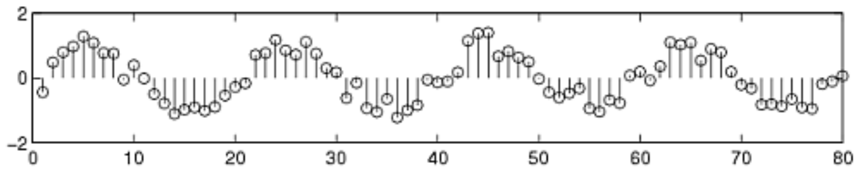
L1.3

Signal Classification (5) – Deterministic vs Random

Deterministic



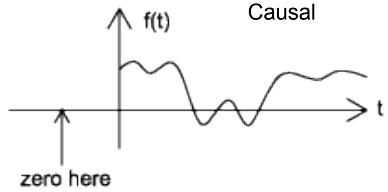
Random



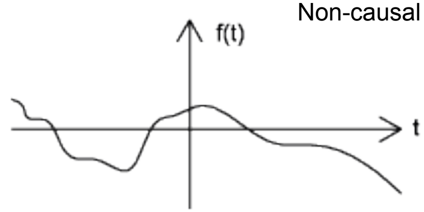
L1.3

Signal Classification (6) – Causal vs Non-causal

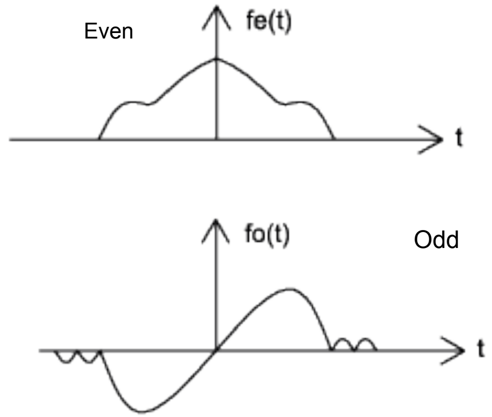
Causal



Non-causal



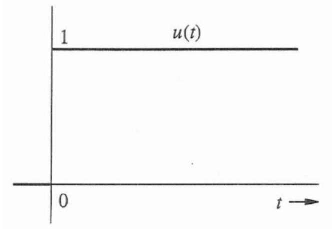
Signal Classification (7) – Even vs Odd



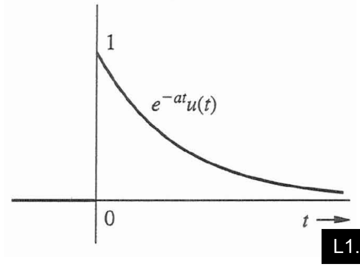
Signal Models (1) – Unit Step Function *u(t)*

- Step function defined by:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



- Useful to describe a signal that begins at t = 0 (i.e. causal signal).
- For example, the signal e^{-at} represents an everlasting exponential that starts at t = -∞.
- The causal for of this exponential can be described as: $e^{-at}u(t)$

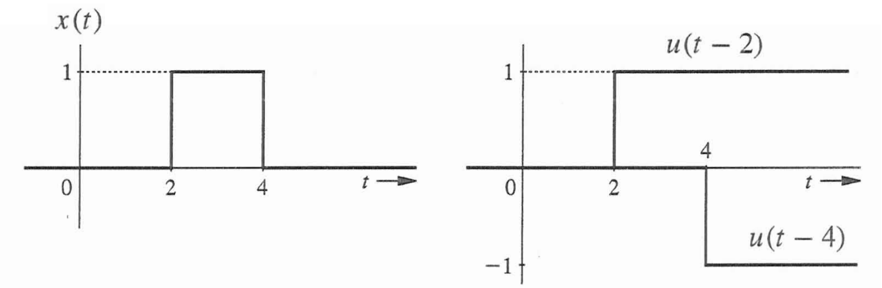


L1.4.1

Signal Models (2) – Pulse signal

- A pulse signal can be presented by two step functions:

$$x(t) = u(t-2) - u(t-4)$$



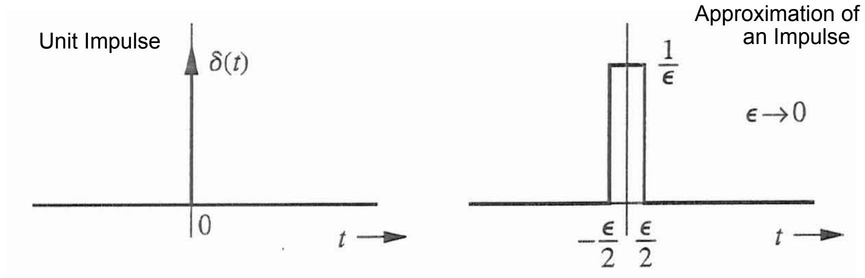
L1.4.1

Signal Models (3) – Unit Impulse Function δ*(t)*

- First defined by Dirac as:

$$\delta(t) = 0 \qquad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

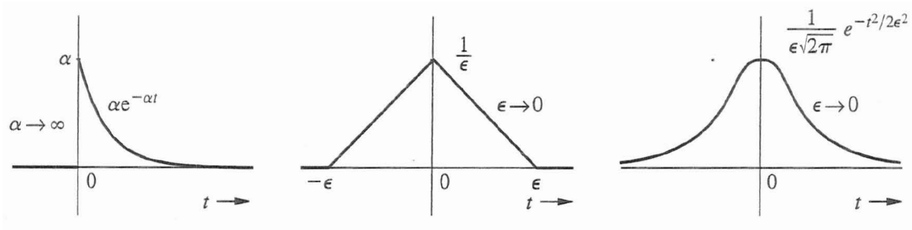


L1.4.2

Signal Models (4) – Unit Impulse Function δ(*t*)

- May use functions other than a rectangular pulse. Here are three example functions:
- Note that the area under the pulse function must be unity

Exponential | Triangular | Gaussian



L1.4.2

Multiplying a function Φ(t) by an Impulse

- Since impulse is non-zero only at t = 0, and Φ(t) at t = 0 is Φ(0), we get:

$$\phi(t)\delta(t) = \phi(0)\delta(t)$$

- We can generalise this for t = T:

$$\phi(t)\delta(t-T) = \phi(T)\delta(t-T)$$

L1.4.2

Sampling Property of Unit Impulse Function

- Since we have: $\phi(t)\delta(t) = \phi(0)\delta(t)$
- It follows that:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)\,dt = \phi(0)\int_{-\infty}^{\infty}\delta(t)\,dt = \phi(0)$$

- This is the same as "sampling" Φ(t) at t = 0.
- If we want to sample Φ(t) at t = T, we just multiple Φ(t) with $\delta(t-T)$

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-T)\,dt = \phi(T)$$

- This is called the "sampling or sifting property" of the impulse.

L1.4.2

The Exponential Function e^{st} (1)

- This exponential function is very important in signals & systems, and the parameter *s* is a complex variable given by:

$$s = \sigma + j\omega$$

Therefore

$$e^{st} = e^{(\sigma+j\omega)t} = e^{\sigma t}e^{j\omega t} = e^{\sigma t}(\cos\omega t + j\sin\omega t)$$

Since $s^* = \sigma - j\omega$ (the conjugate of s), then

$$e^{s^*t} = e^{\sigma - j\omega} = e^{\sigma t}e^{-j\omega t} = e^{\sigma t}(\cos\omega t - j\sin\omega t)$$

and

$$e^{\sigma t}\cos\omega t = \tfrac{1}{2}(e^{st} + e^{s^*t})$$

L1.4.3

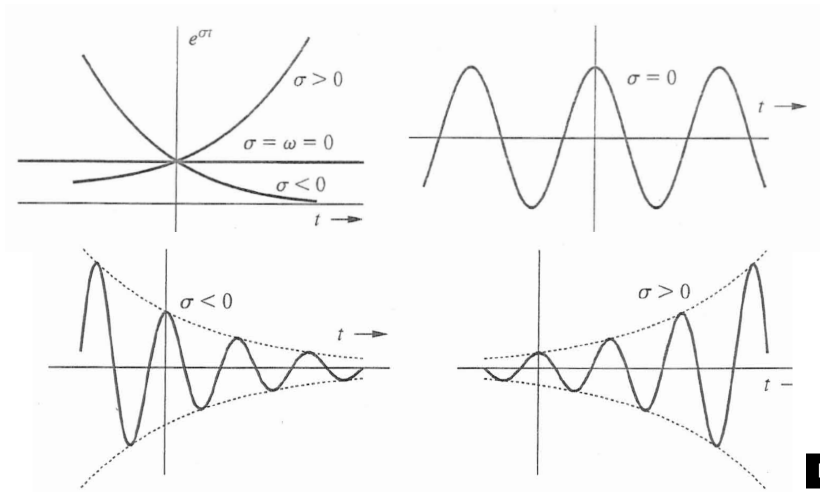
The Exponential Function e^{st} (2)

- If $\sigma = 0$, then we have the function $e^{j\omega t}$, which has a real frequency of ω
- Therefore the complex variable $s = \sigma + j\omega$ is the **complex frequency**
- The function e^{st} can be used to describe a very large class of signals and functions. Here are a number of example:

1. A constant $k = ke^{0t}$ $\quad (s = 0)$
2. A monotonic exponential $e^{\sigma t}$ $\quad (\omega = 0,\ s = \sigma)$
3. A sinusoid $\cos \omega t$ $\quad (\sigma = 0,\ s = \pm j\omega)$
4. An exponentially varying sinusoid $e^{\sigma t} \cos \omega t$ $\quad (s = \sigma \pm j\omega)$

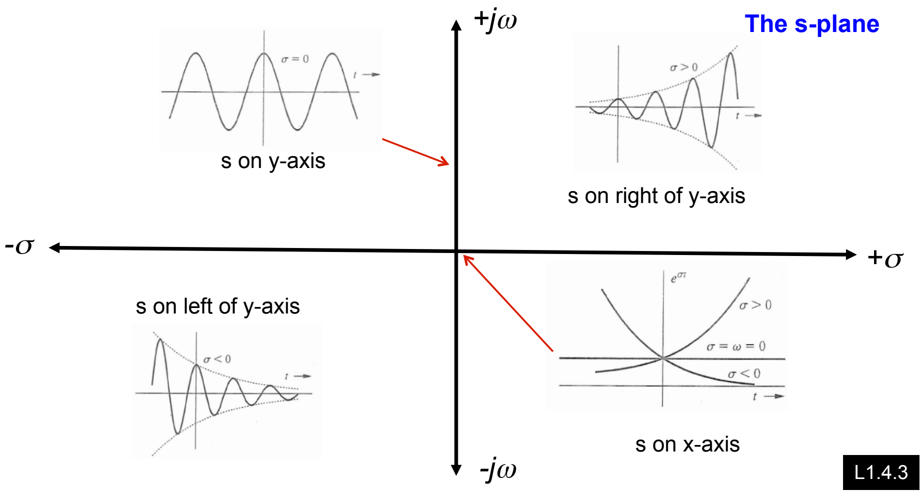
L1.4.3

The Exponential Function e^{st} (2)



L1.4.3

The Complex Frequency Plane $s = \sigma + j\omega$

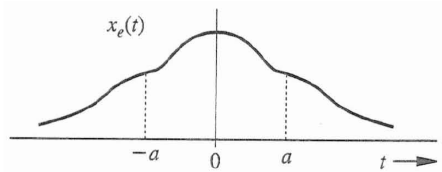


L1.4.3

Even and Odd functions (1)

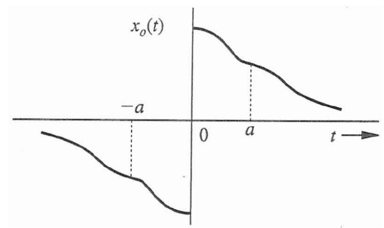
- A real function $x_e(t)$ is said to be an even function of t if

$$x_e(t) = x_e(-t)$$



- A real function $x_o(t)$ is said to be an odd function of t if

$$x_o(t) = -x_o(-t)$$



L1.5

Even and Odd functions (2)

- Even and odd functions have the following properties:
  - Even x Odd = Odd
  - Odd x Odd = Even
  - Even x Even = Even
- Every signal *x(t)* can be expressed as a sum of even and odd components because:

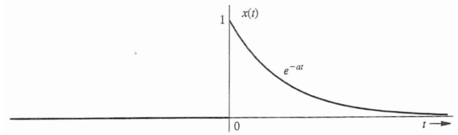
$$x(t) = \underbrace{\tfrac{1}{2}[x(t) + x(-t)]}_{\text{even}} + \underbrace{\tfrac{1}{2}[x(t) - x(-t)]}_{\text{odd}}$$

L1.5

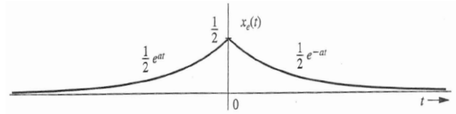
Even and Odd functions (3)

- Consider the causal exponential function $x(t) = e^{-at}u(t)$

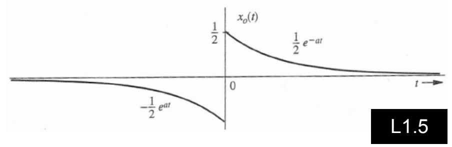
$$x(t) = x_e(t) + x_o(t)$$



$$x_e(t) = \tfrac{1}{2}[e^{-at}u(t) + e^{at}u(-t)]$$



$$x_o(t) = \tfrac{1}{2}[e^{-at}u(t) - e^{at}u(-t)]$$



L1.5

Relating this lecture to other courses

- The first part of this lecture on signals has been covered in this lecture was covered in the 1st year Communications course (lectures 1-3)
- This is mostly an introductory and revision lecture